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# Nonlinear analysis of historic and contemporary vaulted masonry assemblages



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# **KEYWORDS**

Masonry; Nonlinear analysis; Historic structures; Domes; Vaults; Shells; ANSYS **Abstract** This paper incorporates both analytical and experimental investigations of the nonlinear behavior of unreinforced masonry assemblages, especially curved elements such as arches, vaults and domes. The conservation of architectural and cultural heritage necessitates going through a comprehensive scientific procedure of assessment of unreinforced assemblages. Usually, linear analysis is conducted for simplifying analysis and design of masonry structures. However, such simplification might underestimate the structural capacity of these constructions in many cases, and thus the nonlinear analysis gives better description for the actual behavior and capacity of the structure. The present theoretical study utilizes finite element discretization, using a commercial nonlinear analysis computer program (ANSYS), which renders the approach easily and efficiently applicable by a practicing engineer. The adopted solution procedure is explained regarding material characterization and nonlinear solution parameters.

Also, an experimental study was conducted in order to validate the accuracy of the adopted modeling and solution procedure by comparison with experimental results. Validation of the model was also ensured by means of comparison between the calculated numerical results and experimental results available in the literature. Further, the proposed modeling procedure was applied on existing historic and contemporary structures to demonstrate the ability of the proposed analysis to capture the behavior observed in real structures. Applications of the adopted procedure for design of new masonry constructions demonstrated the applicability of the proposed models in engineering practice. Finally, some conclusions and recommendations are presented.

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# Introduction

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Masonry is the oldest yet still widely used construction method. Egypt possesses an enormous wealth of existing ancient and historical monumental structures constructed of stone or brick masonry. The need of conservation of this invaluable architectural and cultural heritage for future generations necessitates going through a comprehensive scientific procedure of assessment of these structures [1]. In addition, there

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is presently an increasing awareness and demand for the use of masonry wall bearing construction system. This construction system has many advantages over the widespread reinforced concrete skeleton system such as economy, durability and sustainability [2]. However, accurate structural analysis of masonry constructions is a true challenge. Being composed of masonry units bonded by mortar, the mechanical behavior of masonry structural elements exhibits non-homogeneity and directional properties, in addition to cracking due to weakness and brittleness of mortar joints. For simplifying the analysis and design of masonry structures, usually linear isotropic behavior is assumed. However, such simplification might underestimate the structural capacity of such constructions in many cases [3]. Therefore, there is a need for availability of accurate yet simple to use numerical tool, which is capable of describing the behavior of the structure from the linear stage, through cracking until complete loss of strength [1].

In this research paper, nonlinear analysis of unreinforced masonry structures is performed through finite element discretization using a commercially-available computer program, ANSYS. This renders the approach applicable by a practicing designer. The adopted solution procedure is explained in the following sections regarding material characterization and nonlinear solution parameters.

In addition, an experimental program was conducted in order to validate the accuracy of the proposed theoretical study. Furthermore, the proposed modeling was applied on existing historic and contemporary structures to demonstrate the ability of the proposed analysis to capture the behavior observed in real structures. Examples of design of new masonry constructions were also performed and serve to demonstrate the applicability of the proposed model in engineering practice. Finally, conclusions are drawn and some recommendations are presented.

# Approaches for modeling and nonlinear analysis

Masonry is a heterogeneous material that consists of units and joints. Units are such as bricks, blocks, ashlars, adobes, irregular stones and others. Mortar can be clay, bitumen, chalk lime/cement based mortar, glue or others. The huge number of possible combinations generated by the geometry, nature and arrangement of units as well as the characteristics of mortars raise doubts about the accuracy of the term "masonry" [4]. The mechanical behavior of the different types of masonry has generally common features: high specific mass, low tensile and shear strengths and low ductility (brittle behavior). The numerical analysis of masonry structures is mostly performed by using the Finite Element Method (FEM). The analysis begins by generating a finite element model of the structural element or the entire structure. In the geometrical model, different elements can be employed to represent columns, arches, domes and vaults such as truss, beam, solid, membrane, plate and/or shell elements. For representing the heterogeneous and anisotropy of masonry construction, it is possible to use different modeling strategies depending on the level of accuracy and the simplicity desired. These strategies, illustrated in Fig. 1, are described below [5].

- (a) Detailed micro-modeling: both mortar masonry units are modeled independently as continuum elements where inelastic properties for each can be assigned. Additionally, discontinuous elements are used to model the interface between mortar and units, as shown in Fig. 1(a). This kind of analysis demands the knowledge of each of the constituents of masonry (unit and mortar) as well as the interface, and should include all the failure mechanisms of masonry, namely, cracking of joints, sliding over one head or bed joint, cracking of the units and crushing of masonry [6]. Applications can be carried out using finite elements, discrete elements or limit analysis [7]. Micro-modeling studies need higher computational effort but give better understanding about the local behavior of masonry structures. In general, this approach is particularly adequate for research and in small models for localized analysis [3,5].
- (b) Simplified micro-modeling: expanded units are represented by continuum elements whereas the behavior of the mortar joints and unit-mortar interface is lumped in discontinuous elements, known as interface elements, as shown in Fig. 1(b). Masonry is thus considered as a set of elastic blocks bonded by potential fracture/slip lines at the joints [8].
- (c) Macro-modeling (homogenization theory): The simplest strategy where masonry units, mortar and mortar-unit interface are smeared out in a homogenous continuum material, as represented in Fig. 1(c). Thus, masonry is treated as a homogenous anisotropic continuum, in which the macro constitutive behavior of masonry is obtained from a mathematical process involving the geometry and the constitutive behavior of the masonry components [9,10]. Macro models are more applicable when the structure has large dimensions and stresses are uniformly distributed along the macro-length [5,9].



Fig. 1 Modeling strategies for masonry structures: (a) detailed micro-modeling; (b) simplified micro-modeling and (c) macro-modeling [5].

This macro-modeling is adopted in the present research work, due to the reduced time and memory requirements as well as a user-friendly mesh generation [1].

#### Numerical analysis

#### Adopted numerical representation and solution procedure

Within this research work, macro-modeling was adopted where masonry is considered as a homogenous anisotropic continuum in which the macro behavior is simulated through the selection of specific material properties [11]. The commercial computer software ANSYS [12] is used for finite element discretization and for nonlinear analysis. Material behavior obeys the vonmises vield criteria coupled with an isotropic work hardening assumption. The solid element SOLID65 is used for 3D structural modeling [12]. It is defined by eight nodes having three translation degrees of freedom at each node. The element formulation is capable of cracking in three orthogonal directions, crushing, plastic deformation and creep, and is therefore suitable for nonlinear material properties. The user defines the material tensile stress, compressive stress, and shear transfer coefficient which range from zero for smooth crack (complete loss of shear transfer) to 1.0 representing a rough crack (no loss of shear transfer). When the solution converges to the cracked state, the modulus and consequently the stiffness normal to the crack face is set to zero. For nonlinear analysis, an iterative solution is adopted with load applied at increments. Within each load step, the computer program may perform several substeps in which equilibrium iterations are made until convergence criteria are satisfied and a converged solution is reached. Linear analysis of some studied cases was performed using the commercial finite element program SAP [13], in order to compare results obtained from linear and nonlinear analyses.

#### Masonry mechanical properties

For nonlinear analysis using ANSYS, the ultimate strength of masonry in compression and tension should be defined. The material uniaxial behavior is described by a multi-linear stress–strain curve starting at the origin, with user-defined stress–strain points. The slope of the first segment corresponds to the material elastic modulus entered by the user. Thus, the mechanical properties of masonry are all governed by the compressive strength.

*Compressive strength:* extensive studies and research work were done concerning the behavior of masonry in compression till failure [14,15]. The ultimate compressive strength of masonry in the direction normal to the bed joints is determined experimentally by compression testing of masonry prisms [17,18]. In design codes and specifications, it is represented by the characteristic compressive strength ( $f_m$ ) [17].

*Tensile strength:* for tensile stresses normal to the bed joints, failure is generally caused by failure of the relatively low tensile bond strength between the mortar bed joint and the masonry unit [4]. Masonry tensile strength may be equated to the tensile bond strength between the joint and the unit, experimentally determined by several researches to be in the range of 10-20% of masonry compressive strength [3,4,16].

Accordingly, values for tensile strength will be assumed in this range for the computer linear and nonlinear analyses.

*Stress-strain relation:* the stress-strain curve can be determined experimentally from the masonry prism compression test by plotting the relation between the stress and accumulated strain in the specimen. The stress-strain relation adopted in this study showed resemblance to the one described in textbooks [4,14].

Modulus of elasticity  $(E_m)$ : the modulus of elasticity  $(E_m)$  is specified as the secant modulus of the stress-strain curve [4]. The Egyptian code of practice suggests a value of 700  $f'_m$  for clay masonry [17].

# Numerical study

The proposed numerical study is performed in order to point out the significance of nonlinear analysis. Unreinforced masonry simple beams having different spans and all having breadth of 400 mm, as illustrated in Fig. 2, are studied under their own weight only. For comparison, numerical modeling of the beams is made by the linear analysis program SAP2000 (v14.1.0) [13] and nonlinear analysis program ANSYS (v.9.1) [12], as shown in Fig. 3. Failure of the beam is assumed to occur when maximum tensile or compressive strength for masonry is reached. Ultimate compressive and tensile strengths for masonry were assumed in the numerical study based on experimental results of local brick units and prisms. For experimental verification of the obtained numerical results, some of the studied beams will be prepared and tested till failure as will be explained in the following section. The minimum required depth for stability of each beam under its own weight is evaluated by several trial runs assuming the maximum limit for masonry tensile and compressive stresses.

#### Results of the numerical study

The results of the proposed numerical study, shown in Table 1 and Fig. 4, give the minimum possible depth H of each beam in case of linear and nonlinear runs. The obtained results emphasize the gap between linear and nonlinear analyses. Nonlinear analysis gives depth less than that calculated by linear analysis by values 56–76%. This is because when tensile stresses at any location exceed the ultimate tensile stress, cracks result. Linear analysis assumes failure of the element at this stage, while non-linear analysis modifies the structure stiffness and therefore redistribution of stresses will take place and the element or structure can even sustain more loads till complete failure [14].

#### **Experimental program**

#### Experimental determination of material properties

Test samples were prepared to evaluate the basic mechanical properties necessary to specify a stress–strain curve for the masonry material to be used in the numerical analysis.

(a) Brick unit test: compression tests were made on local shale brick units of dimensions  $200 \times 100 \times 60$  mm, the test apparatus, shown in Fig. 5, is provided with wooden plates for even stress distribution of compression over the brick face.



Fig. 2 Typical dimensions of the studied masonry beams.



Fig. 3 Numerical modeling: (a) linear SAP model and (b) nonlinear ANSYS model.

Table 1	able 1 Results of the numerical study.							
L (mm)	Minimum poss	ible height for beam (mm)	Difference between linear & nonlinear (%)					
	SAP linear	ANSYS linear	ANSYS nonlinear					
1000	70	70	25	64				
1500	170	150	75	56				
2000	300	250	100	67				
2500	550	500	125	77				
3000	850	800	200	76				
3500	1200	1150	400	67				
4000	1600	1500	700	56				

- (b) Mortar test: compression test was made using mortar cubes with dimensions  $100 \times 100 \times 100$  mm, in accordance with ASTM [19] and using mortar designated as type 2 by the Egyptian code for design of masonry structures [17]. The test apparatus and set up are shown in Fig. 6.
- (c) Prism test: the masonry prism test is recommended by several codes [17,18], to determine the value of the masonry characteristic compressive strength  $(f_m')$ . The tested prisms consisted of five brick units as shown in Fig. 7.
- (d) Shear bond test: the test was made to evaluate the shear bond strength of masonry; the specimen consisted of three brick units bonded by mortar joints as shown in Fig. 8.

The compressive strength for brick unit, mortar cube and prism is given in Table 2 as the average of three tested samples. The shear bond test results showed large variation between the three samples and were thus unreliable. The masonry tensile strength is suggested by several researches to be assumed equal to 0.1-0.2 of the compressive strength [4,15]. Therefore, in the present numerical study, the limiting tensile strength was assumed equal to 0.425 MPa for both the linear and nonlinear computer analysis programs.

# Preparation and testing of masonry beams

Preparation and dimensions of the tested masonry beams Three unreinforced masonry beams were selected to verify experimentally the finite element modeling and solution. The



Fig. 4 Comparison between linear and nonlinear results for the studied masonry beams.

beams were built using local shale bricks and cement mortar designated as mortar 2 by the Egyptian code [17]. Similar bricks were loosely stacked, covered with foam sheets and used as forms for building the masonry beams, as shown in Fig. 7, and were later removed after 28 days. The dimensions of the three beams sketched in Fig. 2 are listed in Table 3.

# Test procedure

After releasing the masonry form, it was clear that all three beams are stable and safe under their own weight. To evaluate the failure loads for the beams, incremental loads of 500 N-sand packages were placed on the beams middle third position and deflection was measured, as shown in Fig. 8. Failure of beam B occurred when the incrementally applied load reached 2.7 kN.

#### Numerical verification

The experimental failure load for beam B was applied to the numerical model to determine the actual limiting tensile stress. Nonlinear analysis was performed using ANSYS program,



Fig. 5 Compression test for masonry units and mortar cubes.



Fig. 6 Masonry prism test and shear bond test.



Fig. 7 The finished masonry beam A.



Fig. 8 Loading and instrumentation for beam B.

Table 2		
Specimen	Crushing load (kN)	Strength (MPa)
Brick unit	115	5.75
Mortar	110	11.0
Prism	85	4.25

with trials starting with a value for tensile strength of 0.425 MPa, and then increasing this tensile limit till failure occurring at the experimentally determined 2.7 kN, as shown in Fig. 9. The corresponding value for tensile limit was found to be equal to 0.5625 MPa. Using this updated value for tensile strength of 0.565 MPa in the ANSYS model, the predicted additional load causing failure of beams A and C was found to be equal to 5 kN and 5.1 kN, respectively. Experimentally, load steps of 0.5 kN were applied on beams A and C till failure. The failure load was 5 kN, the same value was predicted which verifies the numerical solution.

# Discussion of numerical and experimental results

The experimental and numerical results show that all the beams could resist loading beyond the limit given by linear analysis. This emphasizes the importance of carrying out

Table 3	Dimensions of masonry tested beams.				
Beam ID	Span (mm)	Actual depth of the sample (mm)	ANSYS nonlinear H (mm)		
A	3000	210	200		
В	2000	100	100		
С	1000	100	25		

nonlinear analysis for masonry structures. The numerical and experimental results demonstrated that unreinforced masonry beams having depths less by 56–76% than the minimum depth suggested by linear analysis could sustain their own weights and even carry additional load till failure.

It was demonstrated that the limit for tensile strength determined from the nonlinear analysis model was 0.565 MPa. This limit represents a ratio of 0.13 of compressive strength, which is within the range found in text books [4] and reported by researchers [15] as previously stated. Failure loads of beams A and C numerically predicted using this tensile limit, were verified experimentally.

The Egyptian code [17] allowable tensile stress along bed joints is equal to 0.07 MPa, which is only 12% of the ultimate tensile stress obtained in the experimental results. This low limit for tensile and also compressive stresses specified in ECP [17] underestimates the capacity of masonry structures which may imply a doubting atmosphere and limit the wide application of load-bearing masonry structures in major engineering projects, in spite of all its benefits from the structural, environmental and economical points of view.

### Case studies and applications

The proposed numerical modeling was applied over some of existing historic and contemporary structures in order to demonstrate the capability of the proposed models to describe the behavior of masonry structures.

#### Comparison with published experimental results

The presented models were used to evaluate the stresses and failure mode of an example studied in previously published



Fig. 9 Numerical nonlinear analysis results for beam B.

research [20], in order to verify the adopted modeling and nonlinear solution procedure. An unreinforced masonry test sample  $750 \times 750$  mm, was subjected to diagonal tension test, as shown in Fig. 10(a). Numerical modeling was made for a panel having the same dimensions, material properties and loading conditions as the published research [20]. The published experimental results showed that most of tensile strain was concentrated within a small band near the center of the panel [20]. Results of the present numerical nonlinear analysis by ANSYS show good agreement with the published results, as seen in Fig. 11. Also, the crack pattern obtained by the proposed model resembles the test sample failure mode, as shown in Fig. 10.

# Assessment of a historic structure

Application of the adopted numerical representation was made on one of the most famous medieval masonry structures, Qalawun Madrassa situated at El-Muizz Street in old Cairo. A comprehensive structural investigation was conducted through a national conservation project in order to outline the causes of the problem and suggest the proper strengthening and rehabilitation schemes [21] .The present study investigates the interior façade overlooking the central open courtyard shown in Fig. 12(a). The walls, abutments and arches of thickness about 1000 mm are made of large regularly shaped limestone blocks bonded by a mortar of lime, sand and flyash, typical of that era. The masonry mechanical properties were based on compressive strength for the stone material determined from tests carried out through the investigations made during the restoration project. Other values were assumed typical of this type of construction. Material properties were given lower values at locations of observed deterioration. The finite element mesh is shown in Fig. 12(b) and the stresses Sx and Sz obtained from linear and nonlinear analyses are shown in Figs. 13 and 14, respectively.

Results obtained from linear analysis show that the stresses ranged from -3.0 to 0.6 MPa, with the highest compressive stresses occurring in the marble column and the highest tensile stresses are at the tip and springing of the middle level arches. Nonlinear analysis results show that the stresses ranged from -3 to 0.35 MPa, which were entered as limits for compression and tension. The results of linear analysis showed that at the tip of the arch tensile stresses of about 0.6 MPa, which exceeds the ultimate limit specified, meaning that tension cracking at this location is assumed resulting in failure of the element. Nonlinear analysis showed tensile stresses at the same point 0.12 MPa only, as shown in Fig. 14. This is due to the redistribution of the tensile stresses performed by nonlinear analysis, thus the studied element could sustain the applied loads without cracking. The crack pattern shown in Fig. 12(c) proves the good structural state of the arch as no cracks were observed in the real arch, which validates the accuracy of the adopted modeling procedure.

The results of the finite element model and the crack pattern indicate the good structural state of the building, since no tensile stress exceeded the allowable values. The results also show that linear analysis gave values for the stresses that were



Fig. 10 Experimental and numerical results; (a) tested panel [20], (b) failure of sample [20] and (c) crack pattern of present study obtained by ANSYS model.



Fig. 11 Finite element analysis stresses; (a) stresses in published results [20] and (b) stresses obtained in present work.



Fig. 12 Qalawon Madrassa; (a) interior facade, (b) nonlinear analysis model and (c) crack pattern.



Fig. 13 Linear analysis results by SAP (a) Sx and (b) Sz.

very high compared to those obtained by nonlinear analysis. These results lead us to the following conclusions:

- (1) The principle of redistribution of stresses is an important concept for analysis of masonry structures which leads to the safety of the building.
- (2) The allowable stresses specified by the Egyptian design code for tension and compression are very low and underestimate the actual load carrying capacity for these elements.

#### Assessment of contemporary unreinforced masonry building

Application of the adopted modeling and solution procedure was made to study the present structural condition of existing two-story villa constructed using unreinforced brick masonry in 2000. The roof of the reception area is as a barrel vault having span 4300 mm and length 6500 m. The vault and supporting walls are built of masonry clay brick units. The vault in the interior and external façade was in good structural condition, but one longitudinal crack was obvious in the key of the vault, as shown in Fig. 15(a). Nonlinear analysis was carried out as structural assessment of the existing building with the objective of interpreting occurrence of the crack. Material properties were assumed based on typical results for clay masonry units available in the local market. These were taken as: mass density 0.18 ton  $s^2/m^4$ , modulus of elasticity 2975 MPa, major Poisson's ratio 0.15, crushing limit 4.25 MPa, cracking limit 0.425 MPa, shear coefficient 0.2–0.8 for opened and closed crack, respectively. The nonlinear stress–strain curve for the masonry material was defined and entered in the computer model having values based on the compressive strength of the brick unit.

The structure was modeled by finite elements and nonlinear analysis was first carried out in order to check the safety of the vault in the original conditions under its own weight. The obtained results showed that the stresses occurring in the vault were within the allowable values and no cracks were present in the crack pattern of the vault under its own weight. To ac-



Fig. 14 Nonlinear analysis results by ANSYS (a) Sx and (b) Sz.



Fig. 15 The observed crack and the finite element mesh for vault.

count for occurrence of the crack, several loading cases were tried by introducing differential settlement of supports at different locations with several assumed values; the finite element model is shown in Fig. 15(b). The vault was cracked vertically at the key of the vault and also above the key of the supporting arch, so differential settlement can be assumed to have occurred in the abutment of the vault. The results for the settlement assumption show excessive tensile stresses at the tip of the vault and at the key of the supporting arch, also the crack pattern resulting from the numerical analysis was the same as observed in the existing vault, as shown in Fig. 16.

The nonlinear analysis results demonstrated the stability and safety of the vault under its own weight, but when differential settlement was introduced, results showed cracks of the same location and pattern observed in the actual vault. This result gives interpretation for occurrence of the observed cracks and helps in suggestion of repair proposal. The results prove the capability of the adopted numerical modeling to describe the structural behavior of the studied case.

### Application to design of new masonry constructions

The main problem faced by the design of masonry structures through linear analysis is that tension stresses usually exceed the masonry tensile stresses allowed by most design codes. Usually, wall thickness will be increased or steel reinforcement will be provided at these locations. However, evidence proves that these structures are quite safe, as similar masonry structures are observed to survive for very long ages and not collapse or even show visible cracks. This may be attributed to the fact that the stresses are redistributed within the structural element after the tensile stresses reach the limiting value for masonry. Thus, a nonlinear analysis that allows for stress redistribution is more realistic for describing the actual behavior of unreinforced masonry structures.

The objective of this study is to demonstrate the efficiency and ease of application of the adopted numerical modeling to reliable design of new structures made of load bearing masonry elements. The proposed model is applied for the



Fig. 16 Nonlinear analysis results; (a) stresses Sx and (b) crack pattern.



Fig. 17 Finite element analysis results; (a) SAP results and (b) ANSYS results.

design of a structure already designed and constructed within a major project of the Ismaili Center in Dubai [22].

The studied structure is composed of a dome built using brick units and supported on seven stone arches. During the design stages of the project, finite element modeling and linear analysis were carried out using SAP2000 program. The resulting tensile stresses in the dome and supporting arches, shown in Fig. 17(a), exceeded the allowable stresses of the ECP code [17], so it was decided to use steel reinforcement and concrete grout in order to satisfy code design requirements.

In the present study, nonlinear analysis was carried out for the same structure using ANSYS program assuming the same material properties. Results obtained from the nonlinear analysis under own weight, shown in Fig. 17(b), indicate the presence of tensile stresses and several tensile cracks at the crown and base of the studied arches. Still, these tensile cracks do not cause failure of the structure. To determine the capacity of the structure, loads were increased gradually in the numerical model. Failure of the structure occurred when loads were increased by about 35%. This result eliminates the use of steel reinforcement which was previously suggested based on previous results from linear analysis. This will render the construction simpler and more economic. This case study demonstrates the capability of the proposed nonlinear analysis to represent efficiently the stress redistribution occurring within the masonry structure.

#### **Conclusions and recommendations**

This paper incorporated both analytical and experimental investigation of the nonlinear behavior of unreinforced masonry assemblages. From the experimental and theoreticalnumerical studies, the following main conclusions may be presented.

- The adopted modeling and nonlinear solution were done using a commercially available computer program (ANSYS), which renders the approach applicable by a practicing engineer.
- (2) The obtained experimental and numerical results show that unreinforced masonry beams having depths less by 56–76% than the minimum depth suggested by linear analysis could sustain their own weights and even carry additional weights till failure. The Egyptian code [17] prohibits the use of unreinforced masonry beams for new construction. Also, tensile strength in unreinforced masonry is not allowed for permanent loading cases [17]. This emphasizes the importance of carrying out nonlinear analysis for masonry structures.
- (3) Application of the proposed modeling procedure on existing historic and contemporary structures demonstrated the ability of the model to capture the behavior of masonry assemblages having complex three-dimensional geometries and curved elements such as arches, vaults and domes. In addition, the proposed numerical modeling is proved suitable to study and understand the structural behavior of existing heritage structures and interpret the cracks or any structural problem encountered in it.
- (4) The ultimate capacity of wall bearing masonry structures is considerably under-estimated if linear analysis is carried out. Nonlinear analysis gives a much better representation of the structural behavior of masonry elements regarding ultimate capacity and cracking pattern.
- (5) The limit for masonry tensile strength determined experimentally and numerically was 0.565 MPa, representing a ratio of 0.13 of compressive strength, which is within the range found in text books and reported by researchers.
- (6) The low limit for compression or tension stresses stated in most design codes underestimates the actual load carrying capacity of masonry elements and assemblages. These low values for allowable stresses should be revised in order to encourage the wide application of load-bearing masonry construction in major engineering projects.
- (7) It is recommended that more sophisticated nonlinear plasticity models and elements should be used in future studies for more complex structures.
- (8) It is recommended to conduct more extensive numerical studies using more sophisticated material representation of masonry properties, in order to reach comprehensive explanation for masonry brittleness. Identification and proper definition of this property can drive code developers to use strict estimates of the strength to avoid brittle failure.

# **Conflict of interest**

None declared.

# References

- P.B. Lourenço, Structural restoration of monuments: recommendations and advances in research and practice, in: Proceedings of 1st International Conference on Restoration of Heritage Masonry Structures, April 24–27, Supreme Council of Antiquities, Cairo, Egypt, 2006.
- [2] A.A. Hamid, How to face the increasing cost of reinforcing steel in RC frame construction in Egypt, Keynote lecture, in: Workshop of Proposed Alternatives to Face the Increasing Cost of Steel Reinforcement Used in RC Structures in Egypt, Housing and Building Research Center, Cairo, Egypt, 2006 (January 2).
- [3] A. Giordano, E. Mele, A. De Luca, Modelling of historical masonry structures: comparison of different approaches through a case study, Eng. Struct. 24 (8) (2002) 1057–1069.
- [4] R.E. Klingner, Masonry Structural Design, McGraw-Hill Professional, 2010, p. 588.
- [5] P.B. Lourenço, Computational strategies for masonry structures (Ph.D. thesis), Delft University, The Netherlands, 1996.
- [6] L. Macorini, B.A. Izzuddin, Nonlinear analysis of masonry structures using mesoscale partitioned modeling, Adv. Eng. Software 60–61 (2013) 58–69.
- [7] A. Orduña, P.B. Lourenço, Three-dimensional limit analysis of rigid blocks assemblages. Part II: load-path following solution procedure and validation, Int. J. Solids Struct. 42 (18–19) (2005) 5161–5180.
- [8] G. Giambanco, S. Rizzo, R. Spallino, Numerical analysis of masonry structures via interface models, Comput. Methods Appl. Mech. Eng. 190 (49–50) (2001) 6493–6511.
- [9] S.Y. Chen, F.L. Moon, T. Yi, A macroelement for the nonlinear analysis of in-plane unreinforced masonry piers, Eng Struct. Elsevier 30 (8) (2008) 2242–2252.
- [10] G. Milani, P.B. Lourenço, A. Tralli, Homogenization approach for the limit analysis of out-of-plane loaded masonry walls, J. Struct. Eng. ASCE 132 (10) (2006) 1650–1663.
- [11] T.S. El-Salakawy, Analysis and design of load-bearing masonry structures (M.Sc. thesis), Faculty of Engineering at Shobra, Benha University, 2009.
- [12] ANSYS Nonlinear Analysis Computer Program, Release 9.1, Theory Reference Manual, ANSYS Inc., Canonsburg, PA, USA, 2008.
- [13] Sap2000, Advanced14.1.0, Computer Software for Static and Dynamic Finite Element Analysis of Structures, Computers & Structures Inc., California, USA, 2009.
- [14] H.N. Nguyen, A Study on the Mechanical Behavior and Modeling of Masonry, Master of Engineering Report, Department of Civil and Environmental Engineering, University of California, Berkeley, USA, 2003.
- [15] J.L. Pina-Henriques, Masonry under compression: failure analysis and long-term effects (Ph.D. thesis), University of Minho, Portugal, 2005.
- [16] A.A. Hamid, A.C. Heidebrecht, Tensile strength of concrete masonry, J. Struct. Div. ASCE 105 (7) (1979) 1261–1276.
- [17] ECP204-2005, Egyptian Code for Design of Masonry Structures, Housing and Building Research Center, Ministry of Housing Utilities and Urban Communities, Egypt, 2005.
- [18] American Society for Testing and Materials, ASTM C 1314-03b, Standard Test Method for Compressive Strength of Masonry Prisms, ASTM International, West Conshohocken, PA, 2004.
- [19] American Society for Testing and Materials, ASTM C 270-04a, Standard Specification for Mortar for Unit Masonry, ASTM International, West Conshohocken, PA, 2004.
- [20] K.M. Mosalam, Retrofitting of unreinforced masonry walls using glass fiber reinforced polymer laminates, in: Proceedings

of Conference on Future Vision and Challenges for Urban Development, Dec. 20–22, Housing and Building Research Center, Cairo, Egypt, 2004.

[21] A.M. Elleboudy, G.A. Hamdy, Strengthening and Repair of a Monument in Cairo, in: First European Workshop on Structural Health Monitoring, SHM2002, ENS-Cachan, Paris, 2002 (July10–12).

[22] R. Eldahan, M.I. Saafan, Low cost housing, in: Workshop on Economic Challenges to Reduce Building Costs, Housing and Building Research Center, Cairo, Egypt, 2008.